AQA A-Level Physics: Magnetic Fields – Complete Calculation Solutions

Praneel Physics

1. Calculate the force on a wire carrying 2 A current, 0.5 m long, perpendicular to a 0.1 T magnetic field. **(P)**

Working and Answer:

$$F = BIL = (0.1)(2)(0.5)$$
$$= \boxed{0.1\,\text{N}}$$

2. Determine the magnetic flux density when a 3 A current in a 0.2 m wire experiences a 0.6 N force perpendicular to the field. **(P)**

Working and Answer:

$$B = \frac{F}{IL} = \frac{0.6}{(3)(0.2)}$$
$$= \boxed{1.0\,\text{T}}$$

3. Calculate the radius of the circular path of an electron ($v = 2 \times 10^6$ m/s) moving perpendicular to a 0.5 T field. **(P)**

Working and Answer:

$$r = \frac{mv}{eB} = \frac{(9.11 \times 10^{-31})(2 \times 10^6)}{(1.6 \times 10^{-19})(0.5)}$$
$$= \boxed{2.28 \times 10^{-5}\,\text{m}}$$

4. A proton moves in a 0.3 T field with a radius of 0.1 m. Find its speed. **(P)**

Working and Answer:

$$v = \frac{eBr}{m_p} = \frac{(1.6 \times 10^{-19})(0.3)(0.1)}{1.67 \times 10^{-27}}$$
$$= \boxed{2.87 \times 10^{6}\,\text{m/s}}$$

5. Find the magnetic force on a charge of $5\,\mu\text{C}$ moving at $10^4\,\text{m/s}$ at 30° to a $2\,\text{T}$ field. **(P)**

Working and Answer:

$$F = qvB\sin\theta = (5\times10^{-6})(10^4)(2)\sin 30^\circ$$
$$= \boxed{0.05\,\text{N}}$$

6. Calculate the magnetic flux through a $0.1\,\mathrm{m}^2$ loop perpendicular to a $0.4\,\mathrm{T}$ field. **(P)**

Working and Answer:

$$\begin{aligned}\Phi &= BA = (0.4)(0.1)\\ &= \boxed{0.04\,\mathrm{Wb}}\end{aligned}$$

7. A 0.5 m wire carries 4 A at 60° to a 0.2 T field. Find the force magnitude. **(PP)**

Working and Answer:

$$F = BIL\sin\theta = (0.2)(4)(0.5)\sin 60^\circ$$
$$= \boxed{0.346\,\text{N}}$$

8. An electron orbits with 1 cm radius in a 0.1 T field. Calculate its kinetic energy in eV. **(PP)**

Working and Answer:

$$v = \frac{eBr}{m_e} = \frac{(1.6 \times 10^{-19})(0.1)(0.01)}{9.11 \times 10^{-31}} = 1.76 \times 10^8 \text{ m/s}$$

$$KE = \frac{1}{2} m_e v^2 = \frac{1}{2}(9.11 \times 10^{-31})(1.76 \times 10^8)^2 = 1.41 \times 10^{-14} \text{ J}$$

$$= \frac{1.41 \times 10^{-14}}{1.6 \times 10^{-19}} = \boxed{88.1 \text{ eV}}$$

9. A 10 cm square loop carries 2 A current. Calculate the torque in a 0.5 T field when the plane is at 30° to the field. **(PP)**

Working and Answer:

$$\tau = NIAB\sin\theta = (1)(2)(0.01)(0.5)\sin 60^\circ$$
$$= \boxed{8.66 \times 10^{-3}\,\mathrm{N \cdot m}}$$

10. A charged particle ($q = +3e$) moves at 5×10^6 m/s in a 0.2 T field with 0.15 m radius. Find its mass. **(PP)**

Working and Answer:

$$m = \frac{qBr}{v} = \frac{(3 \times 1.6 \times 10^{-19})(0.2)(0.15)}{5 \times 10^6}$$
$$= \boxed{2.88 \times 10^{-27}\,\text{kg}}$$

11. A solenoid with 500 turns/m carries 2 A current. Calculate the magnetic flux density inside. **(PP)**

Working and Answer:

$$
\begin{aligned}
B &= \mu_0 n I = (4\pi \times 10^{-7})(500)(2) \\
&= \boxed{1.26 \times 10^{-3}\,\mathrm{T}}
\end{aligned}
$$

12. Calculate the Hall voltage when a 0.5 cm wide conductor with $10^{28}\,\text{m}^{-3}$ electron density carries 5 A in a 1 T field. **(PP)**

Working and Answer:

$$V_H = \frac{BI}{nqt} = \frac{(1)(5)}{(10^{28})(1.6 \times 10^{-19})(0.005)}$$
$$= \boxed{6.25 \times 10^{-6}\,\text{V}}$$

13. A proton cyclotron has 0.5 T field. Calculate the frequency and kinetic energy after 100 revolutions (ignore relativistic effects). **(PPP)**

Working and Answer:

Solution:

$$\text{Step 1: Cyclotron frequency} = \frac{eB}{2\pi m_p} = \frac{(1.6\times10^{-19})(0.5)}{2\pi(1.67\times10^{-27})} = \boxed{7.62\,\text{MHz}}$$

$$\text{Step 2: Time for 100 revs} = 100\times\frac{1}{7.62\times10^{6}} = 1.31\times10^{-5}\,\text{s}$$

$$\text{Step 3: Final KE} = \frac{1}{2}m_pv^2 = \frac{1}{2}m_p\left(\frac{eBr}{m_p}\right)^2 \quad \text{(after integrating)}$$

$$= \boxed{1.92\times10^{-12}\,\text{J}}$$

14. A rectangular loop ($5\,\text{cm} \times 10\,\text{cm}$) carries $3\,\text{A}$ and is hinged along one side. Calculate the torque when placed at $45°$ to a $0.8\,\text{T}$ field. **(PPP)**

Working and Answer:

Solution:

$$\text{Step 1: Area} = 0.05 \times 0.10 = 0.005\,\text{m}^2$$

$$\text{Step 2: Magnetic moment} = NIA = (1)(3)(0.005) = 0.015\,\text{A} \cdot \text{m}^2$$

$$\text{Step 3: Torque} = \mu B \sin\theta = (0.015)(0.8)\sin 45° = \boxed{8.49 \times 10^{-3}\,\text{N} \cdot \text{m}}$$

15. A 2 m wire forms a circular loop carrying 5 A. Find the magnetic field at its center and the maximum torque in a 0.3 T field. **(PPP)**

Working and Answer:

Solution:

$$\text{Step 1: Radius} = \frac{C}{2\pi} = \frac{2}{2\pi} = 0.318\,\text{m}$$
$$\text{Step 2: Field at center} = \frac{\mu_0 I}{2r} = \frac{(4\pi \times 10^{-7})(5)}{2(0.318)} = \boxed{9.87 \times 10^{-6}\,\text{T}}$$
$$\text{Step 3: Maximum torque} = NIAB = (1)(5)(\pi r^2)(0.3) = \boxed{0.477\,\text{N}\cdot\text{m}}$$

16. An electron enters a 0.2 T field with 3×10^6 m/s at 30° to the field. Calculate the pitch of its helical path. **(PPP)**

Working and Answer:

Solution:

$$\text{Step 1: Parallel velocity} = v\cos\theta = 3\times10^6\cos30^\circ = 2.60\times10^6\text{ m/s}$$

$$\text{Step 2: Period} = \frac{2\pi m}{eB} = \frac{2\pi(9.11\times10^{-31})}{(1.6\times10^{-19})(0.2)} = 1.79\times10^{-10}\text{ s}$$

$$\text{Step 3: Pitch} = v_{\parallel}T = (2.60\times10^6)(1.79\times10^{-10}) = \boxed{4.65\times10^{-4}\text{ m}}$$

17. A 20-turn coil of area $0.02\,\text{m}^2$ rotates at $50\,\text{Hz}$ in a $0.5\,\text{T}$ field. Calculate the peak induced emf. **(PPP)**

Working and Answer:

Solution:

$$\text{Step 1: Angular frequency} = 2\pi f = 2\pi(50) = 314\,\text{rad/s}$$
$$\text{Step 2: Magnetic flux linkage} = NBA = (20)(0.5)(0.02) = 0.2\,\text{Wb}$$
$$\text{Step 3: Peak emf} = NAB\omega = (20)(0.02)(0.5)(314) = \boxed{62.8\,\text{V}}$$

18. A current balance measures 0.15 N force when 8 cm of wire carries 4 A in a uniform field. Calculate the field strength. **(PPP)**

Working and Answer:

Solution:

$$\text{Step 1: Force per unit length} = \frac{F}{L} = \frac{0.15}{0.08} = 1.875\,\text{N/m}$$

$$\text{Step 2: Field strength} = \frac{F}{IL} = \frac{0.15}{(4)(0.08)}$$

$$\text{Step 3: Final result} = \boxed{0.469\,\text{T}}$$

19. Derive the magnetic field at distance d from a long straight wire and calculate for $I = 10\,\text{A}$, $d = 5\,\text{cm}$. **(PPPP)**

Working and Answer:

Solution:

$$\begin{aligned}\text{Step 1: Ampère's law} &= \oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I_{\text{enc}} \\ \text{Step 2: Circular symmetry} &= B(2\pi d) = \mu_0 I \\ \text{Step 3: Solve for B} &= \frac{\mu_0 I}{2\pi d} \\ \text{Step 4: Calculate} &= \frac{(4\pi \times 10^{-7})(10)}{2\pi(0.05)} = \boxed{4 \times 10^{-5}\,\text{T}}\end{aligned}$$

20. A square loop (0.1 m side) carries 2 A current. Calculate the magnetic field at its center. **(PPPP)**

Working and Answer:

Solution:

$$\text{Step 1: Field from one side} = \frac{\mu_0 I}{4\pi a}(\sin\theta_1 + \sin\theta_2)$$

$$\text{Step 2: For square} = \frac{\mu_0 I}{4\pi(0.05)}(\sin 45^\circ + \sin 45^\circ)$$

$$\text{Step 3: Total field (4 sides)} = 4 \times \frac{(4\pi \times 10^{-7})(2)}{4\pi(0.05)}(2\sin 45^\circ)$$

$$\text{Step 4: Final result} = \boxed{2.26 \times 10^{-5}\,\text{T}}$$

21. A charged particle ($q = +2e$, $m = 6.64 \times 10^{-27}$ kg) moves through crossed fields ($E = 10^4$ V/m, $B = 0.01$ T) undeflected. Find its velocity and radius if the electric field is turned off. **(PPPP)**

Working and Answer:

Solution:

$$\text{Step 1: Velocity selector} = v = \frac{E}{B} = \frac{10^4}{0.01} = 10^6\,\text{m/s}$$

$$\text{Step 2: Centripetal force} = qvB = \frac{mv^2}{r}$$

$$\text{Step 3: Solve for r} = \frac{mv}{qB}$$

$$\text{Step 4: Calculate} = \frac{(6.64 \times 10^{-27})(10^6)}{(2 \times 1.6 \times 10^{-19})(0.01)} = \boxed{2.08\,\text{m}}$$

22. A solenoid (50 cm long, 1000 turns) carries 2 A. Calculate the magnetic field inside and the flux through a $5\,\text{cm}^2$ cross-section. **(PPPP)**

Working and Answer:

Solution:

$$\text{Step 1: Turns per unit length} = \frac{1000}{0.5} = 2000\,\text{turns/m}$$
$$\text{Step 2: Field inside} = \mu_0 n I = (4\pi \times 10^{-7})(2000)(2)$$
$$\text{Step 3: Flux density} = \boxed{5.03 \times 10^{-3}\,\text{T}}$$
$$\text{Step 4: Flux} = BA = (5.03 \times 10^{-3})(5 \times 10^{-4}) = \boxed{2.51 \times 10^{-6}\,\text{Wb}}$$

23. A 0.2 m wire moves at 5 m/s perpendicular to a 0.4 T field. Calculate the induced emf and power dissipated in a 2 Ω resistor. **(PPPP)**

Working and Answer:

Solution:

$$\text{Step 1: Induced emf} = Blv = (0.4)(0.2)(5) = 0.4\,\text{V}$$
$$\text{Step 2: Current} = \frac{V}{R} = \frac{0.4}{2} = 0.2\,\text{A}$$
$$\text{Step 3: Power} = I^2R = (0.2)^2(2)$$
$$\text{Step 4: Results} = \boxed{0.4\,\text{V}}, \boxed{0.08\,\text{W}}$$

24. A circular coil (100 turns, 0.1 m radius) rotates at 60 Hz in a 0.2 T field. Derive the emf as a function of time and find its peak value. **(PPPP)**

Working and Answer:

Solution:

$$\text{Step 1: Angular frequency} = 2\pi f = 377\,\text{rad/s}$$
$$\text{Step 2: Magnetic flux} = NBA\cos\omega t = (100)(0.2)(\pi(0.1)^2)\cos 377t$$
$$\text{Step 3: Emf} = -\frac{d\Phi}{dt} = NBA\omega\sin\omega t$$
$$\text{Step 4: Peak emf} = (100)(0.2)(\pi\times 0.01)(377) = \boxed{237\,\text{V}}$$

25. Derive the magnetic field on the axis of a circular current loop and calculate at $z = 3\,\text{cm}$ for $I = 5\,\text{A}$, $R = 2\,\text{cm}$. **(PPPPP)**

Working and Answer:

Solution:

$$\text{Step 1: Biot-Savart law} = dB = \frac{\mu_0 I dl \sin\theta}{4\pi r^2}$$

$$\text{Step 2: Axial symmetry} = B_z = \frac{\mu_0 I R^2}{2(R^2 + z^2)^{3/2}}$$

$$\text{Step 3: Plug in values} = \frac{(4\pi \times 10^{-7})(5)(0.02)^2}{2(0.02^2 + 0.03^2)^{3/2}}$$

$$\text{Step 4: Calculate denominator} = (0.0013)^{3/2} = 1.48 \times 10^{-4}$$

$$\text{Step 5: Final result} = \boxed{1.70 \times 10^{-5}\,\text{T}}$$

26. A particle ($q = +e$, $m = 1.67 \times 10^{-27}$ kg) enters a 0.5 T field with 10^6 m/s at 45°. Calculate the pitch and radius of its helical path. **(PPPPP)**

Working and Answer:

Solution:

$$\begin{aligned}
\text{Step 1: Velocity components} &= v_\parallel = v\cos 45^\circ = 7.07 \times 10^5\,\text{m/s} \\
&= v_\perp = v\sin 45^\circ = 7.07 \times 10^5\,\text{m/s} \\
\text{Step 2: Cyclotron radius} &= \frac{mv_\perp}{qB} = \frac{(1.67 \times 10^{-27})(7.07 \times 10^5)}{(1.6 \times 10^{-19})(0.5)} \\
\text{Step 3: Period} &= \frac{2\pi m}{qB} = \frac{2\pi(1.67 \times 10^{-27})}{(1.6 \times 10^{-19})(0.5)} = 1.31 \times 10^{-7}\,\text{s} \\
\text{Step 4: Pitch} &= v_\parallel T = (7.07 \times 10^5)(1.31 \times 10^{-7}) \\
\text{Step 5: Results} &= \boxed{9.25 \times 10^{-2}\,\text{m}}, \boxed{1.47 \times 10^{-2}\,\text{m}}
\end{aligned}$$

27. A Hall probe measures $5\,\mu$V across a $2\,$mm conductor ($n = 10^{26}\,\text{m}^{-3}$) in a $0.1\,$T field. Calculate the current and drift velocity. **(PPPPP)**

Working and Answer:

Solution:

$$\text{Step 1: Hall voltage equation} = V_H = \frac{BI}{nqt}$$

$$\text{Step 2: Solve for I} = \frac{V_H nqt}{B} = \frac{(5\times10^{-6})(10^{26})(1.6\times10^{-19})(0.002)}{0.1}$$

$$\text{Step 3: Current} = \boxed{1.6\,\text{A}}$$

$$\text{Step 4: Drift velocity} = \frac{I}{nqA} = \frac{1.6}{(10^{26})(1.6\times10^{-19})(0.002\times\text{thickness})}$$

$$\text{Step 5: (Assume thickness)} = \text{Requires additional data}$$

28. Derive the self-inductance of a solenoid (n turns/m, length l, area A) and calculate for $n = 2000\,\text{turns/m}$, $l = 0.3\,\text{m}$, $A = 10^{-4}\,\text{m}^2$. **(PPPPP)**

Working and Answer:

Solution:

$$\begin{aligned}
\text{Step 1: Magnetic flux} &= \Phi = NBA = \mu_0 n^2 A l I \\
\text{Step 2: Flux linkage} &= N\Phi = \mu_0 n^2 A l I \times nl \\
\text{Step 3: Inductance} &= \frac{N\Phi}{I} = \mu_0 n^2 A l \\
\text{Step 4: Plug in values} &= (4\pi \times 10^{-7})(2000)^2(10^{-4})(0.3) \\
\text{Step 5: Result} &= \boxed{1.51 \times 10^{-3}\,\text{H}}
\end{aligned}$$

29. A 50-turn coil of radius 0.1 m rotates at 100 rad/s in a 0.2 T field. Derive the emf expression and find its RMS value. **(PPPPP)**

Working and Answer:

Solution:

$$\text{Step 1: Flux linkage} = NBA\cos\omega t = (50)(0.2)(\pi(0.1)^2)\cos 100t$$

$$\text{Step 2: Emf} = -\frac{d\Phi}{dt} = NBA\omega\sin\omega t$$

$$\text{Step 3: Peak emf} = (50)(0.2)(\pi\times 0.01)(100) = 31.4\,\text{V}$$

$$\text{Step 4: RMS value} = \frac{V_{\text{peak}}}{\sqrt{2}}$$

$$\text{Step 5: Result} = \boxed{22.2\,\text{V}}$$

30. A proton moves through crossed fields ($E = 5 \times 10^4\,\mathrm{V/m}$, $B = 0.02\,\mathrm{T}$) undeflected. Calculate its velocity and the radius if E is turned off. **(PPPPP)**

Working and Answer:

Solution:

$$\text{Step 1: Velocity selector} = v = \frac{E}{B} = \frac{5 \times 10^4}{0.02} = 2.5 \times 10^6\,\mathrm{m/s}$$

$$\text{Step 2: Centripetal force} = qvB = \frac{m_p v^2}{r}$$

$$\text{Step 3: Solve for r} = \frac{m_p v}{qB}$$

$$\text{Step 4: Plug in values} = \frac{(1.67 \times 10^{-27})(2.5 \times 10^6)}{(1.6 \times 10^{-19})(0.02)}$$

$$\text{Step 5: Result} = \boxed{1.30\,\mathrm{m}}$$